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$N = \frac{1}{2}$ supersymmetric four-dimensional non-linear σ-models from non-anti-commutative superspace [1]

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Abstract

The component structure of a generic N=1/2 supersymmetric Non-Linear Sigma-Model (NLSM) defined in the four-dimensional (Euclidean) Non-Anti-Commutative (NAC) superspace is investigated in detail. The most general NLSM is described in terms of arbitrary Kähler potential, and chiral and anti-chiral superpotentials. The case of a single chiral superfield gives rise to splitting of the NLSM potentials, whereas the case of several chiral superfields results in smearing (or fuzziness) of the NLSM potentials, while both effects are controlled by the auxiliary fields. We eliminate the auxiliary fields by solving their algebraic equations of motion, and demonstrate that the results are dependent upon whether the auxiliary integrations responsible for the fuzziness are performed before or after elimination of the auxiliary fields. There is no ambiguity in the case of splitting, i.e. for a single chiral superfield. Fully explicit results are derived in the case of the N=1/2 supersymmetric NAC-deformed CP^n NLSM in four dimensions. Here we find another surprise that our results differ from the N=1/2 supersymmetric CP^n NLSM derived by the quotient construction from the N=1/2 supersymmetric NAC-deformed gauge theory. We conclude that an N=1/2 supersymmetric deformation of a generic NLSM from the NAC superspace is not unique.

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1 Introduction

Investigation of various aspects of Non-Anti-Commutative (NAC) superspace and related deformations of supersymmetric field theories are the subject of intensive studies during the last two years (see, e.g., refs. [1, 2, 3, 4, 5, 6, 7, 8] directly related to our title, and the references therein for the earlier work in the NAC-deformed N=1 superspace). Our general motivation is to enhance understanding of the role of spacetime in supersymmetry, while keeping globally supersymmetric field theory under control. Yet additional motivation is provided by string theory, where the non-anti-commutativity can be related to a constant (self-dual) gravi-photon background (see e.g., ref. [1] and references therein).

This paper is devoted to the NAC-deformed supersymmetric Non-Linear Sigma-Models (NLSM) in four dimensions. The bosonic NLSM we consider are the most general scalar field theories, *without* any gauge fields or higher derivatives. Their (un-deformed) N=1 supersymmetric extension is well known to require Kähler geometry of the NLSM metric and a holomorphic scalar superpotential (see e.g., the pioneering paper [9] and ref. [10] for a review).

We work in four-dimensional Euclidean [6] N=1 superspace $(x^\mu,\theta^\alpha,\bar{\theta}^{\dot{\alpha}})$, and use the standard notation [11]. The NAC deformation is given by

$$\{\theta^\alpha,\theta^\beta\}_* = C^{\alpha\beta} \quad , \tag{1.1}$$

where $C^{\alpha\beta}$ are some constants. The remaining superspace coordinates in the chiral basis ($y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$, $\mu,\nu = 1,2,3,4$ and $\alpha,\beta,\ldots = 1,2$) still (anti)commute,

$$[y^\mu,y^\nu] = \{\bar{\theta}^{\dot{\alpha}},\bar{\theta}^{\dot{\beta}}\} = \{\theta^\alpha,\bar{\theta}^{\dot{\beta}}\} = [y^\mu,\theta^\alpha] = [y^\mu,\bar{\theta}^{\dot{\alpha}}] = 0 \ . \tag{1.2}$$

The $C^{\alpha\beta}\neq 0$ explicitly break the four-dimensional 'Lorentz' invariance at the fundamental level. The NAC nature of θ's can be fully taken into account by using the Moyal-Weyl-type (star) product of superfields [1] ,

$$f(\theta) * g(\theta) = f(\theta)\,\exp\left(-\frac{C^{\alpha\beta}}{2}\frac{\overleftarrow{\partial}}{\partial\theta^\alpha}\frac{\vec{\partial}}{\partial\theta^\beta}\right)g(\theta) \ , \tag{1.3}$$

which respects the N=1 superspace chirality. The star product (1.3) is polynomial in the deformation parameter ,

$$f(\theta) * g(\theta) = fg + (-1)^{\deg f}\frac{C^{\alpha\beta}}{2}\frac{\partial f}{\partial\theta^\alpha}\frac{\partial g}{\partial\theta^\beta} - \det C\frac{\partial^2 f}{\partial\theta^2}\frac{\partial^2 g}{\partial\theta^2} \quad , \tag{1.4}$$

[6] The use of Atiyah-Ward spacetime of the signature $(+,+,-,-)$ is another possibility [12].

where we have used the identity

$$\det C = \tfrac{1}{2}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}C^{\alpha\beta}C^{\gamma\delta} \ , \tag{1.5}$$

and the notation

$$\frac{\partial^2}{\partial\theta^2} = \tfrac{1}{4}\varepsilon^{\alpha\beta}\frac{\partial}{\partial\theta^\alpha}\frac{\partial}{\partial\theta^\beta} \quad . \tag{1.6}$$

We also use the following book-keeping notation for 2-component spinors:

$$\theta\chi = \theta^\alpha\chi_\alpha \ , \quad \bar{\theta}\bar{\chi} = \bar{\theta}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} \ , \quad \theta^2 = \theta^\alpha\theta_\alpha \ , \quad \bar{\theta}^2 = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}. \tag{1.7}$$

The spinorial indices are raised and lowered by the use of two-dimensional Levi-Civita symbols [11]. Grassmann integration amounts to Grassmann differentiation. The anti-chiral covariant derivative in the chiral superspace basis is $\bar{D}_{\dot{\alpha}} = -\partial/\partial\bar{\theta}^{\dot{\alpha}}$. The field component expansion of a chiral superfield Φ reads

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\chi(y) + \theta^2 M(y) \quad . \tag{1.8}$$

An anti-chiral superfield $\overline{\Phi}$ in the chiral basis is given by

$$\begin{aligned}\overline{\Phi}(y^\mu - 2i\theta\sigma^\mu\bar{\theta},\bar{\theta}) = \ & \bar{\phi}(y) + \sqrt{2}\bar{\theta}\bar{\chi}(y) + \bar{\theta}^2\bar{M}(y) \\ & + \sqrt{2}\theta\left(i\sigma^\mu\partial_\mu\bar{\chi}(y)\bar{\theta}^2 - i\sqrt{2}\sigma^\mu\bar{\theta}\partial_\mu\bar{\phi}(y)\right) + \theta^2\bar{\theta}^2\Box\bar{\phi}(y) \ ,\end{aligned} \tag{1.9}$$

where $\Box = \partial_\mu\partial_\mu$. The bars over fields serve to distinguish between the 'left' and 'right' components that are truly independent in Euclidean spacetime.

The non-anticommutativity $C_{\alpha\beta} \neq 0$ also explicitly breaks *half* of the original N=1 supersymmetry [1]. Only the chiral subalgebra generated by the chiral supercharges (in the chiral basis) $Q_\alpha = \partial/\partial\theta^\alpha$ is preserved, with $\{Q_\alpha, Q_\beta\}_* = 0$, thus defining what is now called N=1/2 supersymmetry. The use of the NAC-deformed superspace allows one to keep N=1/2 supersymmetry manifest. The N=1/2 supersymmetry transformation laws of the chiral and anti-chiral superfield components in eqs. (1.8) and (1.9) are as follows:

$$\delta\phi = \sqrt{2}\varepsilon^\alpha\chi_\alpha \quad , \quad \delta\chi_\alpha = \sqrt{2}\varepsilon_\alpha M \ , \quad \delta M = 0 \ , \tag{1.10}$$

and

$$\delta\bar{\phi} = 0 \ , \quad \delta\bar{\chi}^{\dot{\alpha}} = -i\sqrt{2}(\tilde{\sigma}_\mu)^{\dot{\alpha}\beta}\varepsilon_\beta\partial_\mu\bar{\phi} \ , \quad \delta\bar{M} = -i\sqrt{2}\partial_\mu\bar{\chi}_{\dot{\alpha}}(\tilde{\sigma}_\mu)^{\dot{\alpha}\beta}\varepsilon_\beta \ , \tag{1.11}$$

respectively, where we have introduced the N=1/2 supersymmetry (chiral) parameter ε^α.

The most general four-dimensional N=1 supersymmetric NLSM action (without any gauge fields) is given by

$$S[\Phi,\overline{\Phi}] = \int d^4y \left[\iint d^2\theta d^2\bar{\theta}\, K(\Phi^i,\overline{\Phi}^{\bar{j}}) + \int d^2\theta\, W(\Phi^i) + \int d^2\bar{\theta}\, \overline{W}(\overline{\Phi}^{\bar{j}}) \right] \ . \qquad (1.12)$$

This action is completely specified by the Kähler superpotential $K(\Phi,\overline{\Phi})$, the scalar superpotential $W(\Phi)$, and the anti-chiral superpotential $\overline{W}(\overline{\Phi})$, in terms of some number n of chiral and anti-chiral superfields, $i,\bar{j} = 1,2,\ldots,n$. In Euclidean superspace the functions $W(\Phi)$ and $\overline{W}(\overline{\Phi})$ are independent upon each other.

The problem of computing the NAC-deformed extension of eq. (1.12) in four dimensions, after a 'Seiberg-Witten map' (i.e. after evaluating all the star products), was solved in ref. [4] in the case of a single (anti)chiral superfield. That solution was extended to the case of several (anti)chiral superfields in ref. [7]. In both cases the perturbative solutions were found, i.e. in terms of the infinite sums with respect to the deformation parameter and the auxiliary fields. A similar problem in two dimensions was perturbatively solved in ref. [2]. A non-perturbative solution (i.e. in a closed form, in terms of finite functions) was found in ref. [6] in two dimensions, and in ref. [8] in four dimensions. The calculations behind each result appear to be quite extensive, so it is non-trivial to make a comparison. In our opinion, an explicit summation is necessary anyway, both for comparison and elimination of the auxiliary fields. Getting the results in a closed form is also crucial for any physical applications of the NAC-deformation and its geometrical interpretation, as well as in investigating concrete examples (see below). In this paper we use our four-dimensional results [8] that are in precise agreement with the basis formulae of ref. [6], after dimensional reduction to two dimensions. The non-perturbative results of refs. [6] and [8] presumably amount to the full summation of the perturbative results in refs. [2] and [4, 7], respectively.

We use the chiral basis that is most suitable for computing NAC-deformation, by reducing the most non-trivial problem of calculation of the NAC-deformed Kähler superpotential to that for the NAC-deformed scalar superpotential [8]. The simple general results about the NAC-deformed scalar superpotentials were found in refs. [5, 6]. Our major concern in this paper is getting solutions to the auxiliary fields that control splitting or fuzziness in the NLSM target space.

Our paper is organized as follows. In sect. 2 we summarize the earlier results [5, 8] that represent our setup here. In sect. 3 we eliminate the auxiliary fields in a generic NAC-deformed NLSM, by using implicit functions. The case of a single (anti)chiral superfield is also considered separately. In sect. 4 we specify our results to the case of the CP^n NLSM. The CP^1 case is fully investigated. Sect. 5 is our conclusion.

2 The NAC Kähler potential and superpotentials

We use the following notation valid for any function $F(\phi, \bar{\phi})$:

$$F_{,i_1 i_2 \cdots i_s \bar{p}_1 \bar{p}_2 \cdots \bar{p}_t} = \frac{\partial^{s+t} F}{\partial \phi^{i_1} \partial \phi^{i_2} \cdots \phi^{i_s} \partial \bar{\phi}^{\bar{p}_1} \partial \bar{\phi}^{\bar{p}_2} \cdots \partial \bar{\phi}^{\bar{p}_t}} \quad , \tag{2.1}$$

and the Grassmann integral normalisation $\int d^2\theta \, \theta^2 = 1$. The actual deformation parameter, in the case of the NAC-deformed field theory (1.12), appears to be

$$c = \sqrt{- \det C} \ , \tag{2.2}$$

where we have used the definition [1]

$$\det C = \tfrac{1}{2} \varepsilon_{\alpha\gamma} \varepsilon_{\beta\delta} C^{\alpha\beta} C^{\gamma\delta} \quad . \tag{2.3}$$

As a result, unlike the case of the NAC-deformed supersymmetric gauge theories [1], the NAC-deformation of the NLSM field theory (1.12) appears to be 'Lorentz'-invariant [5, 8].

A simple non-perturbative formula, describing an arbitrary NAC-deformed scalar superpotential V depending upon a single chiral superfield Φ, was found in ref. [5],

$$\int d^2\theta \, V_*(\Phi) = \tfrac{1}{2c} \left[V(\phi + cM) - V(\phi - cM) \right] - \frac{\chi^2}{4cM} \left[V_{,\phi}(\phi + cM) - V_{,\phi}(\phi - cM) \right] \ . \tag{2.4}$$

The NAC-deformation in the single superfield case thus gives rise to the split of the scalar potential, which is controlled by the auxiliary field M. When using an elementary identity

$$f(x+a) - f(x-a) = a \frac{\partial}{\partial x} \int_{-1}^{+1} d\xi \, f(x + \xi a) \ , \tag{2.5}$$

valid for any function f, we can rewrite eq. (2.4) to the equivalent form, as in ref. [6],

$$\int d^2\theta \, V_*(\Phi) = \tfrac{1}{2} M \frac{\partial}{\partial \phi} \int_{-1}^{+1} d\xi \, V(\phi + \xi c M) - \frac{1}{4} \chi^2 \frac{\partial^2}{\partial \phi^2} \int_{-1}^{+1} d\xi \, V(\phi + \xi c M) \ , \tag{2.6}$$

Similarly, in the case of several chiral superfields, one finds [6]

$$\int d^2\theta \, V_*(\Phi^I) = \tfrac{1}{2} M^I \frac{\partial}{\partial \phi^I} \widetilde{V}(\phi, M) - \frac{1}{4} (\chi^I \chi^J) \frac{\partial^2}{\partial \phi^I \partial \phi^J} \widetilde{V}(\phi, M) \ , \tag{2.7}$$

in terms of the auxiliary pre-potential $\widetilde{V}$ [6],

$$\widetilde{V}(\phi, M) = \int_{-1}^{+1} d\xi \, V(\phi^I + \xi c M^I) \ . \tag{2.8}$$

Hence the NAC-deformation of a generic scalar superpotential V results in its smearing or fuzziness controlled by the auxiliary fields M^I [6].

A calculation of the NAC deformed Kähler potential

$$\int d^4y\, L_{\rm kin.} \equiv \int d^4y \int d^2\theta d^2\bar{\theta}\, K(\Phi^i, \overline{\Phi}^{\bar{j}})_* \tag{2.9}$$

can be reduced to eqs. (2.4) or (2.7), when using a chiral reduction in superspace, with the following result [8]:

$$\begin{aligned} L_{\rm kin.} \;=\;& \tfrac{1}{2}M^iY_{,i} + \tfrac{1}{2}\partial^\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}}Z_{,\bar{p}\bar{q}} + \tfrac{1}{2}\Box\bar{\phi}^{\bar{p}}Z_{,\bar{p}} - \tfrac{1}{4}(\chi^i\chi^j)Y_{,ij} \\ & - \tfrac{1}{2}i(\chi^i\sigma^\mu\bar{\chi}^{\bar{p}})\partial_\mu\bar{\phi}^{\bar{q}}Z_{,i\bar{p}\bar{q}} - \tfrac{1}{2}i(\chi^i\sigma^\mu\partial_\mu\bar{\chi}^{\bar{p}})Z_{,i\bar{p}}\ , \end{aligned} \tag{2.10}$$

where we have introduced the (component) smeared Kähler pre-potential

$$Z(\phi,\bar{\phi},M) = \int_{-1}^{+1} d\xi\, K^\xi \quad \text{with} \quad K^\xi \equiv K(\phi^i + \xi c M^i, \bar{\phi}^{\bar{j}})\ \ , \tag{2.11}$$

as well as the extra (auxiliary) pre-potential

$$Y(\phi,\bar{\phi},M,\bar{M}) = \bar{M}^{\bar{p}}Z_{,\bar{p}} - \tfrac{1}{2}(\bar{\chi}^{\bar{p}}\bar{\chi}^{\bar{q}})Z_{,\bar{p}\bar{q}} + c\int_{-1}^{+1} d\xi\xi \left[\partial^\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}}K^\xi_{,\bar{p}\bar{q}} + \Box\bar{\phi}^{\bar{p}}K^\xi_{,\bar{p}}\right]\ , \tag{2.12}$$

as in ref. [6]. It is not difficult to check that eq. (2.10) does reduce to the standard (Kähler) N=1 supersymmetric NLSM [9] in the limit $c \to 0$. Also, in the case of a free (bilinear) Kähler potential $K = \delta_{i\bar{j}}\Phi^i\bar{\Phi}^{\bar{j}}$, there is no deformation at all.

The NAC-deformed scalar superpotentials $W(\Phi)_*$ and $\bar{W}(\bar{\Phi})_*$ imply, via eqs. (2.7) and (2.8), that the following component terms are to be added to eq. (2.10):

$$L_{\rm pot.} = \tfrac{1}{2}M^i\widetilde{W}_{,i} - \tfrac{1}{4}(\chi^i\chi^j)\widetilde{W}_{,ij} + \bar{M}^{\bar{p}}\bar{W}_{,\bar{p}} - \tfrac{1}{2}(\bar{\chi}^{\bar{p}}\bar{\chi}^{\bar{q}})\bar{W}_{,\bar{p}\bar{q}}\ . \tag{2.13}$$

where we have introduced the smeared scalar pre-potential [6]

$$\widetilde{W}(\phi,M) = \int_{-1}^{+1} d\xi\, W(\phi^i + \xi c M^i)\ \ . \tag{2.14}$$

The anti-chiral scalar superpotential terms are *inert* under the NAC-deformation [4, 5, 6, 7, 8].

The ξ-integrations in the equations above represent the smearing effects. However, the smearing is merely apparent in the case of a single chiral superfield, which gives rise to the splitting (2.4) only. This can also be directly demonstrated from eq. (2.10) when using the identity (2.5) together with the related identity [8]

$$f(x+a) + f(x-a) = \int_{-1}^{+1} d\xi\, f(x+\xi a) + a\frac{\partial}{\partial x}\int_{-1}^{+1} d\xi\,\xi f(x+\xi a)\ . \tag{2.15}$$

The single superfield case thus appears to be special, so that a sum of eq. (2.10) and (2.13) can be rewritten to the bosonic contribution [8]

$$
\begin{aligned}
L_{\text{bos.}} = &+ \tfrac{1}{2}\partial^\mu\bar{\phi}\partial_\mu\bar{\phi}\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \tfrac{1}{2}\Box\bar{\phi}\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{\bar{M}}{2c}\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{2c}\left[W(\phi+cM)-W(\phi-cM)\right]+\bar{M}\frac{\partial\bar{W}}{\partial\bar{\phi}} \quad ,
\end{aligned}
\tag{2.16}
$$

supplemented by the following fermionic terms [8]:

$$
\begin{aligned}
L_{\text{ferm.}} = &- \frac{1}{4c}\bar{\chi}^2\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{i}{2cM}(\chi\sigma^\mu\bar{\chi})\partial_\mu\bar{\phi}\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{i}{2cM}(\chi\sigma^\mu\partial_\mu\bar{\chi})\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{\bar{M}}{4cM}\chi^2\left[K_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{1}{4M}\chi^2\partial^\mu\bar{\phi}\partial_\mu\bar{\phi}\left[K_{,\phi\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\phi\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{4cM^2}\chi^2\partial^\mu\bar{\phi}\partial_\mu\bar{\phi}\left[K_{,\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{1}{4M}\chi^2\Box\bar{\phi}\left[K_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{4cM^2}\chi^2\Box\bar{\phi}\left[K_{,\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&+ \frac{1}{8cM}\chi^2\bar{\chi}^2\left[K_{,\phi\bar{\phi}\bar{\phi}}(\phi+cM,\bar{\phi})-K_{,\phi\bar{\phi}\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&- \frac{1}{4cM}\chi^2\left[W_{,\phi}(\phi+cM)-W_{,\phi}(\phi-cM)\right]-\tfrac{1}{2}\bar{\chi}^2\bar{W}_{,\bar{\phi}\bar{\phi}} \ .
\end{aligned}
\tag{2.17}
$$

The anti-chiral auxiliary fields $\bar{M}^{\bar{p}}$ enter the action (2.10) linearly (as Lagrange multipliers), while their algebraic equations of motion,

$$
\tfrac{1}{2}M^i Z_{,i\bar{p}} - \tfrac{1}{4}(\chi^i\chi^j)Z_{,ij\bar{p}} + \bar{W}_{,\bar{p}} = 0 \ , \tag{2.18}
$$

represent the non-linear set of equations on the auxiliary fields $M^i = M^i(\phi,\bar{\phi})$. [7] As a result, the bosonic scalar potential in components is given by [8]

$$
V_{\text{scalar}}(\phi,\bar{\phi}) = \tfrac{1}{2}M^i\widetilde{W}_{,i}\Big|_{M=M(\phi,\bar{\phi})} \quad . \tag{2.19}
$$

[7] Equation (2.18) is not a linear system because the function Z is M-dependent — see eq. (2.11).

3 Solving for the auxiliary fields in a generic case

The NAC-deformed NLSM in sect. 2 is completely specified by a Kähler function $K(\Phi,\bar{\Phi})$, a chiral function $W(\Phi)$, an anti-chiral function $\bar{W}(\bar{\Phi})$ and a constant deformation parameter c. As a matter of fact, we didn't really used the constancy of c, so our results in sect. 2 are still valid even for a coordinate-dependent NAC deformation $c(y)$. The very possibility of such extension preserving N=1/2 supersymmetry was conjectured in ref. [13] and further investigated in ref. [14]. In this section we assume, however, that $c = const$ for simplicity.

A NAC-deformation is only possible in Euclidean spacetime. However, we may try to press further, by analytically continue our results into Minkowski spacetime (this would require $\det C < 0$). Then the natural place for possible physical applications of NAC-deformation could be given by confining supersymmetric gauge field theories whose low-energy (IR) effective action takes the form of eq. (1.12) in terms some colorless composite scalar superfields Φ and $\bar{\Phi}$ known as the glueball superfields. The use of Konishi anomaly equations [15] allows one to construct exact effective superpotentials of the glueball superfields in various N=1 supersymmetric quantum gauge theories with some number of colors and flavors (see e.g., ref. [16] for details), generalizing the standard Veneziano-Yankielowicz effective potential [17]. This may provide some natural input for the NAC deformation presumably describing some supergravitational corrections to the low-energy effective field theory, as well as for a possible dynamical supersymmetry breaking (see ref. [5] for some attempts in this direction). [8] From that physical point of view, the fact that the NAC-deformation gives rise to a 'Lorentz'-invariant action is clearly a positive feature, whereas the apparent non-Hermiticity of the NAC-deformed action is clearly a negative feature. In this paper we are going to keep the chiral and anti-chiral functions to be arbitrary and concentrate on an investigation of the NAC-deformed kinetic terms.

Solving for the auxiliary fields in eq. (2.10) represents not only a technical but also a conceptual problem because of the smearing effects described by the ξ-integrations. To bring the kinetic terms in eqs. (2.10) and (2.16) to the standard NLSM form (i.e. without the second order spacial derivatives), one has to integrate by parts that leads to the appearance of the spacial derivatives of the auxiliary fields. This implies that one has to solve eq. (2.18) *before* integration by parts. Let $M^i = M^i(\phi,\bar{\phi})$ be a solution to eq. (2.18), and let's ignore fermions for simplicity ($\chi^i_\alpha = \bar{\chi}^{\bar{p}}_{\dot{\alpha}} = 0$). Substituting the

[8] The Konishi anomaly in N=1/2 supersymmetry was recently studied in ref. [18].

solution back to the Lagrangian (2.10) and integrating by parts yields

$$
\begin{aligned}
L_{\text{kin.}}(\phi,\bar{\phi}) = &-\tfrac{1}{2}(\partial_\mu\bar{\phi}^{\bar{p}}\partial_\mu\phi^q)\int_{-1}^{+1}d\xi\left[K^{\xi}_{,\bar{p}q}+2c\xi M^i_{,q}K^{\xi}_{,\bar{p}i}+c\xi M^iK^{\xi}_{,\bar{p}iq}+c^2\xi^2M^iK^{\xi}_{,\bar{p}ij}M^j_{,q}\right] \\
&-\tfrac{1}{2}(\partial_\mu\bar{\phi}^{\bar{p}}\partial_\mu\bar{\phi}^{\bar{q}})\int_{-1}^{+1}d\xi\left[2c\xi K^{\xi}_{,\bar{p}i}M^i_{,\bar{q}}+c^2\xi^2M^iK^{\xi}_{,\bar{p}ij}M^j_{,\bar{q}}\right] \quad .
\end{aligned}
\tag{3.1}
$$

It is now apparent that the NAC-deformation does not preserve the original Kähler geometry in eq. (1.12), though the absence of $(\partial_\mu\phi)^2$ terms and the particular structure of various contributions to eq. (3.1) are remarkable. The action (3.1) takes the form of a generic NLSM, being merely dependent upon mixed derivatives of the Kähler function, so that the original Kähler gauge invariance of eq. (1.12),

$$
K(\phi,\bar{\phi})\to K(\phi,\bar{\phi})+f(\phi)+\bar{f}(\bar{\phi}) \ , \tag{3.2}
$$

with arbitrary gauge functions $f(\phi)$ and $\bar{f}(\bar{\phi})$ is still preserved.

However, a problem arises with the ξ-integrations in eq. (3.1), because the fields M^i are no longer independent and, hence, eq. (2.5) cannot be applied. Instead, eq. (2.5) may have to be replaced by a more general identity

$$
\begin{aligned}
\frac{d}{dx}\int_{-1}^{+1}d\xi\, f(x+\xi a(x)) = &\frac{1}{a(x)}\left[f(x+a(x))-f(x-a(x))\right] \\
&+\frac{1}{a(x)}\left[f(x+a(x))+f(x-a(x))\right]\frac{da}{dx} \\
&-\frac{1}{a^2(x)}\left[F(x+a(x))-F(x-a(x))\right]\frac{da}{dx} \ ,
\end{aligned}
\tag{3.3}
$$

where we have defined $F(x)=\int^x f(\xi)d\xi$. We believe, however, that this way of doing is not quite correct from the viewpoint of the original star product that has nothing to do with dynamics. In other words, the elimination of the auxiliary fields and the ξ-integrations do not commute, while, in our opinion, all the ξ-integrations have to be done *before* solving for the auxiliary fields. This procedure will be adopted in our explicit calculations for the CP^n case in the next sect. 4. It is worth mentioning that the ordering problem does not arise in the case of a sinlge chiral superfield, because there is no need for ξ-integrations there (i.e. when smearing is replaced by splitting). In the last case, when using eq. (2.16), we find

$$
\begin{aligned}
L_{\text{kin.}}(\phi,\bar{\phi}) = &-\tfrac{1}{2}\partial_\mu\bar{\phi}\partial_\mu\phi\left[K_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})+K_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right] \\
&-\tfrac{1}{2}\partial_\mu\bar{\phi}\partial_\mu\phi\left[cK_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})-cK_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right]\frac{\partial M}{\partial\phi} \\
&-\tfrac{1}{2}\partial_\mu\bar{\phi}\partial_\mu\bar{\phi}\left[cK_{,\phi\bar{\phi}}(\phi+cM,\bar{\phi})-cK_{,\phi\bar{\phi}}(\phi-cM,\bar{\phi})\right]\frac{\partial M}{\partial\bar{\phi}} \quad ,
\end{aligned}
\tag{3.4a}
$$

where $M(\phi,\bar{\phi})$ is supposed to be a solution to eq. (2.18). Equation (3.4a) can be rewritten to some other equivalent forms, *viz.*

$$L_{\rm kin.}(\phi,\bar{\phi}) = -\tfrac{1}{2}(\partial_\mu\bar{\phi}\partial_\mu\phi)\frac{\partial}{\partial\phi}\left[K_{,\bar{\phi}}(\phi+cM(\phi,\bar{\phi}),\bar{\phi})+K_{,\bar{\phi}}(\phi-cM(\phi,\bar{\phi}),\bar{\phi})\right]$$

$$-\tfrac{1}{2}(\partial_\mu\bar{\phi}\partial_\mu\bar{\phi})\left[cK_{,\phi\bar{\phi}}(\phi+cM(\phi,\bar{\phi}),\bar{\phi})-cK_{,\phi\bar{\phi}}(\phi-cM(\phi,\bar{\phi}),\bar{\phi})\right]\frac{\partial M(\phi,\bar{\phi})}{\partial\bar{\phi}} \quad , \quad (3.4b)$$

or

$$L_{\rm kin.}(\phi,\bar{\phi}) = -\tfrac{1}{2}(\partial_\mu\bar{\phi}\partial_\mu\phi)\frac{\partial}{\partial\phi}\frac{\partial}{\partial\bar{\phi}}\left[K(\phi+cM(\phi,\bar{\phi}),\bar{\phi})+K(\phi-cM(\phi,\bar{\phi}),\bar{\phi})\right]$$

$$+\tfrac{1}{2}(\partial_\mu\bar{\phi}\partial_\mu\phi)\frac{\partial}{\partial\phi}\left[cK_{,\phi}(\phi+cM,\bar{\phi})-cK_{,\phi}(\phi-cM,\bar{\phi})\right]\frac{\partial M(\phi,\bar{\phi})}{\partial\bar{\phi}}$$

$$-\tfrac{1}{2}(\partial_\mu\bar{\phi}\partial_\mu\bar{\phi})\left[cK_{,\phi\bar{\phi}}(\phi+cM(\phi,\bar{\phi}),\bar{\phi})-cK_{,\phi\bar{\phi}}(\phi-cM(\phi,\bar{\phi}),\bar{\phi})\right]\frac{\partial M(\phi,\bar{\phi})}{\partial\bar{\phi}} \quad . \quad (3.4c)$$

For instance, the first line of eq. (3.4c) gives some Kähler-like terms with the NAC-deformed (splitted) Kähler potential, whereas the second and third lines of eq. (3.4c) apparently violate both Kählerian and Hermitian structures of the original (undeformed) NLSM when $c\neq 0$ and $\partial M(\phi,\bar{\phi})/\partial\bar{\phi}\neq 0$ (cf. the last ref. [2]).

Therefore, the NAC deformation of the Kähler kinetic terms in eq. (1.12) amounts to a non-Kählerian and non-Hermitian deformation of the original Kählerian and Hermitian NLSM, which is controlled by the auxiliary field solution to eq. (2.18). In the case of a single chiral superfield, the deformed NLSM metric can be read off from eq. (3.4). In the case of several superfields, the deformed NLSM can be read off from eq. (3.1), when assuming all the ξ-integrations to be performed with the auxiliary fields considered as the parameters or spectators.

Being unable to explicitly solve eq. (2.18) in a generic case, in the next sect. 4 we consider the simplest non-trivial example provided by the CP^n NLSM with an (undeformed) Kähler, Hermitian and symmetric target space.

4 NAC-deformed CP^n NLSM

The CP^n (principal) NLSM is characterized by a Kähler potential

$$K(\phi,\bar{\phi}) = \alpha\,\ln(1+\kappa^{-2}\phi\bar{\phi}) \quad , \qquad (4.1)$$

where we have used the notation $\phi\bar{\phi}\equiv|\phi|^2\equiv\phi^i\bar{\phi}^i$. Due to a Grassmann nature of the fermionic fields, getting explicit results with fermions is possible. However, those

results appear to be cumbersome and not very illuminating. Therefore, we ignore fermions for simplicity in this section, except of the simplest CP^1 case.

Equations (2.18) in the CP^n case read as follows:

$$\left(\phi^i - \frac{(1+\kappa^{-2}\,|\phi|^{\,2})M^i}{\kappa^{-2}P}\right)\left(\frac{1}{1+\kappa^{-2}\,|\phi|^{\,2}+c\kappa^{-2}P} - \frac{1}{1+\kappa^{-2}\,|\phi|^{\,2}-c\kappa^{-2}P}\right)$$
$$+\frac{2c\bar{W}_{,\bar{\phi}^i}}{\alpha\kappa^{-2}} = 0 \quad , \tag{4.2}$$

where we have used the notation $P = M^i\bar{\phi}^i$.

Multiplying eq. (4.2) with $\bar{\phi}^i$ gives rise to a quadratic equation on P,

$$\frac{\alpha\kappa^2 P}{(\kappa^2+|\phi|^{\,2})^2 - c^2P^2} + U = 0 \ , \tag{4.3}$$

where we have introduced yet another notation $U = \bar{\phi}^i\bar{W}_{,\bar{\phi}^i}$.

A solution to eq. (4.3) is given by

$$P = \frac{\alpha\kappa^2 - \sqrt{\alpha^2\kappa^4 + 4U^2c^2(\kappa^2+|\phi|^{\,2})^2}}{2c^2U} \quad , \tag{4.4}$$

where we have chosen the minus sign in front of the square root, in order to assure the existence of the $c\to 0$ limit. In the case of the CP^1 model, eq. (4.4) already gives a solution for $M = \bar{\phi}^{-1}P$ [8].

In the CP^n case with $n > 1$, substituting the solution (4.4) back into eq. (4.2) yields the following result:

$$M^i = \frac{P\bar{\phi}^i}{\kappa^2+|\phi|^{\,2}} - \frac{(\kappa^2+|\phi|^{\,2})^2 - c^2P^2}{\alpha(\kappa^2+|\phi|^{\,2})}U \quad . \tag{4.5}$$

In the anti-commutative limit $c \to 0$, eq. (4.4) yields

$$\lim_{c\to 0} P = -\frac{(\kappa^2+|\phi|^{\,2})^2}{\alpha\kappa^2}\left(\bar{\phi}^i\bar{W}_{,\bar{\phi}^i}\right) \quad . \tag{4.6}$$

In the case of vanishing anti-chiral superpotential, $\bar{W} = 0$, we have $U = P = 0$, so that the *bosonic* terms in the auxiliary field solution also vanish and, hence, there is no deformation of the bosonic CP^n kinetic terms at all. Given an arbitrary anti-chiral superpotential $\bar{W}(\bar{\Phi}) \neq 0$, the deformed CP^n metric is rather complicated (see e.g., the CP^1 result at the end of this section).

To give an explicit example of the NAC-deformed structure of the NLSM *fermionic* terms, let's consider the CP^1 model in the case of vanishing anti-chiral superpotential. Equation (2.18) now reads

$$M\int_{-1}^{+1} d\xi\, K_{,\phi\bar{\phi}}(\phi + c\xi M, \bar{\phi}) - \tfrac{1}{2}\chi^2\int_{-1}^{+1} d\xi\, K_{,\phi\phi\bar{\phi}}(\phi + c\xi M, \bar{\phi}) = 0 \ , \tag{4.7}$$

whose Kähler function is given by eq. (4.1). The integrations over ξ in eq. (4.7) yield

$$M\left[\frac{-\alpha\kappa^2}{cM\bar{\phi}(\kappa^2+\bar{\phi}(\phi+cM))}+\frac{\alpha\kappa^2}{cM\bar{\phi}(\kappa^2+\bar{\phi}(\phi-cM))}\right] \tag{4.8}$$

$$-\tfrac{1}{2}\chi^2\left[\frac{\alpha\kappa^2}{cM(\kappa^2+\bar{\phi}(\phi+cM))^2}-\frac{\alpha\kappa^2}{cM(\kappa^2+\bar{\phi}(\phi-cM))^2}\right]=0\ ,$$

which gives rise to a simple *cubic* equation on M, [9]

$$M^3-(c\bar{\phi})^{-2}(\kappa^2+|\phi|^2)^2M-\frac{\chi^2}{c^2\bar{\phi}}(\kappa^2+|\phi|^2)=0\ . \tag{4.9}$$

The use of Cardano formula for the roots of a cubic equation gives us three solutions as follows:

$$M=\omega^m\left[-\frac{A_0}{2}+\sqrt{\frac{A_0^2}{4}+\frac{A_1^3}{27}}\right]^{1/3}+\omega^{3-m}\left[-\frac{A_0}{2}-\sqrt{\frac{A_0^2}{4}+\frac{A_1^3}{27}}\right]^{1/3}\ , \tag{4.10}$$

where $\omega=(-1+\sqrt{3})/2$, $m=0,1,2$ and

$$\begin{aligned} A_0&=-\frac{\chi^2}{c^2\bar{\phi}}(\kappa^2+|\phi|^2)\ ,\\ A_1&=-(c\bar{\phi})^{-2}(\kappa^2+|\phi|^2)^2\ . \end{aligned} \tag{4.11}$$

However, due to the nilpotency property of fermions, $(\chi^2)^2=0$ in the CP^1 case, [10] eq. (4.10) is greatly simplified to the followong three solutions:

$$M_0=-\frac{\bar{\phi}\chi^2}{\kappa^2+|\phi|^2} \tag{4.12a}$$

and

$$M_{\pm}=\frac{\pm(\kappa^2+|\phi|^2)}{c\bar{\phi}}+\frac{\chi^2\bar{\phi}}{2(\kappa^2+|\phi|^2)}\ . \tag{4.12b}$$

The solution (4.12a) is clearly the same as that in the undeformed case, whereas the other two solutions are singular in the undeformed limit $c\to 0$. We are unaware of any possible physical significance of the singular solutions.

Inserting the solution (4.12a) into eqs. (2.16) and (2.17), and using the nilpotency condition, $(\chi^2)^2=0$, give rise to the standard (undeformed) four-dimensional N=1 supersymmetric CP^1 NLSM in components. It appears to coincide with the result [3] in the CP^1 case, where also no NAC-deformation of the CP^1 supersymmetric NLSM

[9] In the case of a non-vanishing anti-chiral superpotential, one gets a quartic equation.

[10] In the CP^n case, we merely have $(\chi^i\chi^j)^{n+1}=0$.

was discovered, though our NAC-deformation is different from the one considered there (see sect. 5).

It is of interest to get an explicit NAC-deformed NLSM metric in the case of a *non-vanishing* anti-chiral potential, $\bar{W} \neq 0$. Even, in the CP^1 case, it gives rise to a rather complicated metric, when using eqs. (3.4), (4.1) and the auxiliary field solution

$$M = \frac{\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}\right)^2}}{2c^2\kappa^{-2}\bar{\phi}^2\bar{W}_{,\bar{\phi}}} \tag{4.13}$$

that follows from eq. (4.4), where we have used the notation $\bar{W}_{,\bar{\phi}} = \partial\bar{W}/\partial\bar{\phi}$. A straightforward calculation (both 'by hand' and by the use of a Mathematica computer program) in the CP^1 case yields the following deformed NLSM kinetic terms:

$$L_{\text{kin.}} = -g_{\phi\phi}\partial_\mu\phi\partial_\mu\phi - 2g_{\phi\bar{\phi}}\partial_\mu\phi\partial_\mu\bar{\phi} - g_{\bar{\phi}\bar{\phi}}\partial_\mu\bar{\phi}\partial_\mu\bar{\phi} \ , \tag{4.14}$$

where

$$\begin{aligned}
g_{\phi\bar{\phi}} &= \frac{-\alpha\kappa^{-2}c^2\bar{\phi}^2(\bar{W}_{,\bar{\phi}})^2}{\left(-\alpha + \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}\right)^2}\right)\sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}\right)^2}} \ , \\
g_{\phi\phi} &= 0 \ , \\
g_{\bar{\phi}\bar{\phi}} &= \frac{-2\alpha^{-1}c^2(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}}{\left(\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}\right)^2}\right)\sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}\right)^2}} \times \\
&\quad \times \Big[4c^2\bar{\phi}^2(\bar{W}_{,\bar{\phi}})^3(1+\kappa^{-2}\phi\bar{\phi}) \\
&\quad + \alpha\left(\alpha - \sqrt{\alpha^2 + \left(2c\bar{\phi}(1+\kappa^{-2}\phi\bar{\phi})\bar{W}_{,\bar{\phi}}\right)^2}\right)(2\bar{W}_{,\bar{\phi}} + \bar{\phi}\bar{W}_{,\bar{\phi}\bar{\phi}})\Big] \ .
\end{aligned} \tag{4.15}$$

It is worth noticing that $\det g = -(g_{\phi\bar{\phi}})^2$. The most apparent feature $g_{\phi\phi} = 0$ is also valid in the case of a generic NAC-deformed NLSM (in a given parametrization).

5 Conclusion

As is clear from our discussion and explicit examples, the problem of solving for the auxiliary fields is highly non-trivial in the context of NAC-deformed N=1/2 supersymmetric NLSM under consideration. Special care should be exercised when considering this problem with the smearing effects that should be calculated first.

One should also distinguish between a NAC-deformation and N=1/2 supersymmetry. Though the NAC-deformation we considered is N=1/2 supersymmetric, the

former is *stronger* than the latter. Requiring merely N=1/2 supersymmetry of a four-dimensional NLSM would give rise to much weaker restrictions on the NLSM target space, as is pretty obvious from eqs. (1.10) and (1.11).

Some comments are in order about the relation between our results in this paper and those obtained in ref. [3] for the N=1/2 supersymmetric NAC-deformed CP^n NLSM, in the absence of chiral and anti-chiral scalar superpotentials.

As is well known in the theory of NLSM (see e.g., ref. [10]), the so-called quotient construction (or gauging isometries of the 'flat' NLSM target space) can be used to represent some NLSM with homogeneous target spaces as the gauge theories. It was used in ref. [3] to construct the NAC-deformed N=1/2 supersymmetric NLSM in four dimensions with the CP^n target space, by combining the quotient construction and the results of ref. [1] about the NAC-deformed supersymmetric gauge theories. Unlike the NAC-deformation of the non-gauge supersymmetric field theory (1.12) governed by the scalar deformation parameter c, the NAC-deformed N=1/2 supersymmetric field theory [1] has $C_{\alpha\beta}$ as the deformation parameters, while its action is *not* 'Lorentz'-invariant. In addition, there are more auxiliary fields in the supersymmetric quotient construction, while it does not lead to any splitting or smearing of the Kähler potential. As was demonstrated in ref. [3], the quotient construction of the NAC-deformed N=1/2 supersymmetric CP^n model does *not* modify the Kähler geometry at all, while the *only* new term in the deformed Lagrangian takes the form [3]

$$g_{p\bar{q}}g_{r\bar{s}}C^{\alpha\beta}(\sigma^{\mu\nu})_\beta{}^\gamma\chi^p_\alpha\chi^r_\gamma\partial_\mu\bar{\phi}^{\bar{q}}\partial_\nu\bar{\phi}^{\bar{s}} \quad , \tag{5.1}$$

where $g_{p\bar{q}}(\phi,\bar{\phi})$ is the Fubini-Study metric associated with the Kähler potential (4.1). As is clear from eq. (5.1), this deformation is linear in $C^{\alpha\beta}$, while it is not 'Lorentz'-invariant. The quotient construction itself is also limited to the homogeneous NLSM, and it does not allow a scalar superpotential.

Our approach to the NAC-deformed NLSM is very general, while the NAC deformation of a Kähler potential is controlled by the auxiliary fields M^i entering the deformed Kähler potential in the highly non-linear way. In the absence of an anti-chiral superpotential, when requiring the smooth undeformed limit, the bosonic part of the auxiliary field solution vanishes, so that we are left with the undeformed bosonic Kähler potential as well. However, when the auxiliary fields are eliminated, the structure of the fermionic terms in our approach is apparently not the same as that of ref. [3], even modulo possible field redefinitions.

At the same time, the N=1/2 supersymmetry transformation laws are not modified by a NAC-deformation, both in our approach and in ref. [3]. Our conclusion is that

the two methods give rise to the inequivalent N=1/2 supersymmetric extensions of the Kähler NLSM. It does not seem to be very surprising because there are many N=1/2 supersymmetric extensions of the NLSM kinetic terms.

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